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Stable extendibility of $m\tau_n$ over real projective spaces

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ABSTRACT. The purpose of this paper is to study the stable extendibility of the m-times Whitney sum $m\tau_n$ of the tangent bundle $\tau_n = \tau(RP^n)$ of the n-dimensional real projective space RP^n. We determine the dimension N for which $m\tau_n$ is stably extendible to RP^N but is not stably extendible to RP^{N+1} for $m \leq 10$.

1. Introduction

Let X be a space and A its subspace. A t-dimensional real vector bundle ζ over A is said to be *extendible* (respectively *stably extendible*) to X, if there is a t-dimensional real vector bundle over X whose restriction to A is equivalent (respectively stably equivalent) to ζ, that is, ζ is equivalent (respectively stably equivalent) to the induced bundle $i^*\eta$ of a t-dimensional real vector bundle η over X under the inclusion map $i : A \to X$ (cf. [10, p. 20] and [3, p. 273]). Let RP^n denote the n-dimensional real projective space. For a real vector bundle ζ over RP^n, define an integer $s(\zeta)$ by

$$s(\zeta) = \max\{m \mid m \geq n \text{ and } \zeta \text{ is stably extendible to } RP^m\},$$

where we put $s(\zeta) = \infty$ if ζ is stably extendible to RP^m for every $m \geq n$.

The following theorem is known.

THEOREM 1 ([7, Theorem 4.2]). *For the tangent bundle $\tau_n = \tau(RP^n)$ of RP^n,*

$$s(\tau_n) = \infty \quad \textit{if} \;\; n = 1, 3 \textit{ or } 7; \qquad \textit{and} \qquad s(\tau_n) = n \quad \textit{if} \;\; n \neq 1, 3, 7.$$

The purpose of this paper is to study $s(m\tau_n)$ for $m \geq 2$. Our main results are as follows.

We write simply $s(m, n)$ instead of $s(m\tau_n)$.

THEOREM 2. (1) *If $1 \leq n \leq 8$, then $s(2, n) = \infty$.*

(2) *If $n \geq 9$, then $s(2, n) = 2n + 1$.*

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