

Multiple positive solutions for a semipositone fourth-order boundary value problem

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(Received July 16, 2002)

(Revised January 7, 2003)

ABSTRACT. We consider the nonlinear fourth order boundary value problem

$$u^{(4)}(x) = \lambda f(x, u(x), u'(x))$$

$$u(0) = u'(0) = u''(1) = u'''(1) = 0$$

where $f : [0, 1] \times [0, \infty) \times [0, \infty) \rightarrow (-\infty, \infty)$ is continuous with $f(x, u, p) \geq -M$ for some positive constant M . We show the existence and multiplicity of positive solutions by using a fixed point theorem in cones.

1. Introduction

The deformations of an elastic beam are described by a fourth-order two-point boundary value problem [6]. The boundary conditions are given according to the controls at the ends of the beam. For example, the nonlinear fourth order problem

$$\begin{aligned} u^{(4)}(x) &= \lambda f(x, u(x), u'(x)) \\ u(0) &= u'(0) = u''(1) = u'''(1) = 0 \end{aligned} \tag{1.1}$$

describes the deformations of an elastic beam whose one end fixed and the other end free.

The existence of solutions of (1.1) has been studied by Gupta [6]. But to the best of our knowledge, there are no any results concerning the existence of positive solutions of (1.1). In this paper, we will study the existence and multiplicity of positive solutions of (1.1).

2000 *Mathematics Subject Classification.* 34B18

Key words and phrases. Fourth order boundary value problem, positive solution, cone, fixed point.

*Supported by the NSFC (No. 10271095), GG-110-10736-1003, NWNKJXGC-212 and the Foundation of Major Project in Science and Technology of Chinese Education Ministry.