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# Homotopy types of $m$-twisted $\mathbf{CP}^4$'s

*Dedicated to the memory of Prof. Masahiro Sugawara*

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**ABSTRACT.** We study the homotopy type classification problem of $n$ dimensional $m$-twisted complex projective spaces for the case $n = 4$. In particular, we determine the number of homotopy types of $m$-twisted $\mathbf{CP}^4$'s when $m \geq 1$ is an odd integer.

## 1. Introduction

Let $n \geq 2$ be an integer and let $M$ be a simply-connected $2n$ dimensional finite Poincaré complex. For an integer $m \geq 0$, $M$ is called an *$m$-twisted* $\mathbf{CP}^n$ if there is an isomorphism $H_*(M, \mathbf{Z}) \cong H_*(\mathbf{CP}^n, \mathbf{Z})$ with the condition $x_2 \cdot x_2 = mx_4$, where $x_{2k} \in H^{2k}(M, \mathbf{Z}) \cong \mathbf{Z}$ denotes the corresponding generator $(k = 1, 2)$. Any $m$-twisted $\mathbf{CP}^n$ is homotoy equivalent to a CW complex of the form

$$M \simeq S^2 \cup_{m\eta_2} e^4 \cup e^6 \cup \cdots \cup e^{2n-2} \cup e^{2n} \qquad \text{(homotopy equivalence)},$$

and it has the homotopy type of $2n$ dimensional closed topological manifolds ([6]). Let $\mathscr{M}_m^n$ be the set consisting of all the homotopy equivalence classes of $m$-twisted $\mathbf{CP}^n$'s. For example, when $n = 2$, $\mathscr{M}_1^2 = \{[\mathbf{CP}^2]\}$ and $\mathscr{M}_m^2 = \varnothing$ if $m \neq 1$. When $n = 3$, the following result is known:

THEOREM 1.1 ([11] (cf. [4])). *If $m \geq 0$ is an integer,*

$$\operatorname{card}(\mathscr{M}_m^3) = \begin{cases} 1 & \text{if } m \equiv 1 \pmod 2, \\ 3 & \text{if } m \equiv 0 \pmod 2, \end{cases}$$

*where* $\operatorname{card}(V)$ *denotes the cardinal number of a set* $V$.

In general, it is known that $\mathscr{M}_m^{2k+1} \neq \varnothing$ for any $m, k \geq 2$ (cf. [2]), and we have infinitely many non-trivial examples of $m$-twisted $\mathbf{CP}^{2k+1}$'s. On the other

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