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On the rate of convergence for some linear operators

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Abstract. We consider certain linear operators L_n in polynomial weighted spaces of functions of one variable and study approximation properties of these operators, including theorems on the degree of approximation.

1. Introduction

Approximation properties of Szász-Mirakyan operators

(1)
$$S_n(f;x) := e^{-nx}\sum_{k=0}^{\infty}\frac{(nx)^k}{k!}f\left(\frac{k}{n}\right), \qquad x \in R_0 = [0,+\infty),\ n \in N := \{1,2\dots\},$$

in polynomial weighted spaces C_p were examined in [1]. The space C_p, $p \in N_0 := \{0,1,2,\dots\}$, is associated with the weighted function

(2)
$$w_0(x) := 1, \qquad w_p(x) := (1+x^p)^{-1} \qquad \text{if } \ p \geq 1,$$

and consists of all real-valued continuous functions f on R_0 for which $w_p f$ is uniformly continuous and bounded on R_0. The norm on C_p is defined by

(3)
$$\|f\|_p \equiv \|f(\cdot)\|_p := \sup_{x \in R_0} w_p(x)|f(x)|.$$

In [1] there were theorems on the degree of approximation of $f \in C_p$ by operators S_n defined by (1). From these theorems it was deduced that

$$\lim_{n\to\infty} S_n(f;x) = f(x)$$

for every $f \in C_p$, $p \in N_0$, and $x \in R_0$. Moreover, the above convergence is uniform on every interval $[x_1, x_2]$, $x_2 > x_1 \geq 0$.

In this paper by $M_k(\alpha,\beta)$, we shall denote suitable positive constants depending only on indicated parameters α and β.

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