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Stable maps between 2-spheres with a connected fold curve

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Abstract. Stable maps between 2-dimensional spheres, whose fold curve is connected and its image is simple with minimal number of cusps, are classified for every degree $d \geq 2$.

1. Introduction

We deal with the following problem.

Let M and N be connected surfaces and $f : M \to N$ a smooth map. Then is there a map $h : M \to N$ which satisfies the following conditions?

1. *h is a stable map.*
2. *h is homotopic to f.*
3. *h has a connected fold curve.*
4. *The set of critical values of h has the smallest possible number of singular points.*

Furthermore, how many such maps are there?

By Pignoni [8] the form for the set of critical values of such a map h is determined when $N = \mathbf{R}^2$. In this paper we determine the form for the maps $f : S^2 \to S^2$ with $\deg f = d \geq 2$. More precisely, we show that the set of critical values of such a map has $2d$ cusps and no self-intersections. Furthermore we give the number of their right-left equivalence classes. All such stable maps are right-left equivalent in the case of $d = 2$, but not in the case of $d \geq 3$.

The paper is organized as follows. In §2 we define some notions, the *apparent contour*, the *irreducible contour* and so on. We quote a theorem of Quine [9] which will be used in §3 and §4. In §3 we study the form for the maps $f : S^2 \to S^2$ with $\deg f = 2$. Then we prove that all these maps are right-left equivalent by using the result of [2] and the argument of [3]. In §4 we generalize the argument to the case of $\deg f = d \geq 3$.

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