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Semigroups of nonlinear operators and invariant sets

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Let X be a Banach space and A be an operator in X such that $A-\omega I$ is dissipative for some real number ω. Let $D_a(A)$ be the set of those $x \in \overline{D(A)}$ for which there exists a sequence $\{x_n\}$ in $D(A)$ such that $\lim_{n\to\infty} x_n = x$ and $\limsup_{n\to\infty} |||Ax_n||| < +\infty$ (see [9]). In this paper we are concerned with the set $D_a(A)$.

This work is motivated by the papers of Crandall [2] and Bénilan [1]. Assuming that $R(I-\lambda A) \supset \overline{D(A)}$ for sufficiently small positive numbers λ, Crandall defined a set $\hat{D}(A)$, which is called a generalized domain, and showed that $\hat{D}(A)$ coincides with the set of those $x \in \overline{D(A)}$ for which $T(t)x$ is Lipschitz continuous in t on compact t-sets and therefore is invariant under $T(t)$. Here the semigroup $\{T(t); t \geqq 0\}$ on $\overline{D(A)}$ is defined by $T(t)x = \lim_{n\to\infty} (I-(t/n)A)^{-n}x$ for $t \geqq 0$ and $x \in \overline{D(A)}$. An extension of the result was obtained by Bénilan [1]. He defined the set $\hat{D}(A)$ by considering an extension $\hat{A}$ of A and showed, among others, that if $-A$ is a pseudo-generator then a result of Crandall's type holds for the set $\hat{D}(A)$.

In this paper we establish some sufficient conditions in order that A generates a nonlinear semigroup $\{T(t); t \geqq 0\}$ on $\overline{D(A)}$ such that $D_a(A)$ as well as $\hat{D}(A)$ is invariant under $T(t)$ (Theorem 4.3). The set $D_a(A)$ is in general a subset of $\hat{D}(A)$ and it is shown that if $\liminf_{h\to 0+} h^{-1} d(R(I-hA), x) < +\infty$ for every $x \in \hat{D}(A)$, then $D_a(A) = \hat{D}(A)$ (Proposition 2.3). Thus Theorem 4.3 extends some results in [1] and [2]. The set $D_a(A)$ also possesses some interesting properties. We observe, for instance, that $D_a(A)$ is invariant under certain perturbations. The results in sections 2 and 4 will be used in the final section to prove a result on m-dissipativity.

1. Preliminaries

Let X be a real Banach space with norm $\|\cdot\|$. For a subset S of X, we denote by $\bar{S}$ its closure and by $d(S, x)$ the distance from $x \in X$ to S. Let A be an operator in X. By this we mean a multi-valued operator with domain $D(A)$ and range $R(A)$ both contained in X. We often identify A with its graph $\{[x, y] \in X \times X; x \in D(A), y \in Ax\}$. We denote by $\bar{A}$ the closure of A and we say that A is closed if $A = \bar{A}$. For each $x \in D(A)$, we write

$$|||Ax||| = \inf\{\|y\|; y \in Ax\}.$$