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On the exterior Dirichlet problem for semilinear elliptic equations with coefficients unbounded on the boundary

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Introduction

Let a be a fixed positive constant and let $\Omega_b \equiv \{x \in \boldsymbol{R}^N;\ a < |x| < b\}$, where $N \geqq 2$ and b is a positive constant with $a < b$. And we put $\Omega \equiv \Omega_\infty = \lim_{b\to\infty} \Omega_b$. Consider the problem:

$$(*)_b \qquad \Delta u = (|x| - a)^{-\lambda} G(x) u^\beta \quad \text{in} \quad \Omega_b, \qquad u = 0 \quad \text{on} \quad |x| = a,$$

where β is a real constant, λ is a positive constant and $G(x)$ is a locally Hölder continuous function satisfying some conditions stated below. Note that since $\lambda > 0$, the coefficient of u^β is unbounded on the boundary $\partial\Omega$. So, in general, it is not clear that the problem $(*)_b$ has a solution. When $b = \infty$, the problem $(*)_\infty = (*)$ with $\lambda = 0$ has been studied by many authors and various results on the existence and asymptotic behavior as $|x| \to \infty$ of positive solutions have been obtained. Among them we refer to [2, 3, 6–12, 14]. The first aim of this paper is to obtain global positive solutions of $(*)$ belonging to $C^2(\Omega) \cap C(\bar{\Omega})$ under the condition $\lambda < \beta + 1$. We note that the condition $\lambda < \beta + 1$ is necessary for the existence of solutions of $(*)$ when $G(x) = G(|x|)$. More exactly, we show the existence of infinitely many positive solutions of $(*)$ with some growth properties under $\lambda < \beta + 1$ and the integral conditions

$$\int_a^\infty r^{1-\lambda} (\log (r/a))^\beta G^*(r)\, dr < \infty \qquad (N = 2),$$

$$\int_a^\infty r^{1-\lambda} G^*(r)\, dr < \infty \qquad (N \geqq 3),$$

where $G^*(r) = \max_{|x|=r} |G(x)|$.

The second aim is to show that for any given b ($a < b \leqq \infty$) there exists a solution $u(x)$ of $(*)_b$ belonging to $C^2(\Omega_b)$ which blows up (when $b = \infty$, we say that it grows up.), that is $u(x) \to +\infty(|x| \to b)$, when $\beta > 1$ and $G(x) > 0$, $x \in \Omega_b$.

Our plan in this paper is as follows. In Section 1, we construct global