

# THE FLOW OF WEIGHTS ON FACTORS OF TYPE III

ALAIN CONNES AND MASAMICHI TAKESAKI\*

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**Introduction.** The recent developments in the theory of operator algebras showed the importance of the study of weights. A weight  $\varphi$  on a von Neumann algebra  $M$  is a linear map from  $M_+$  to  $[0, +\infty]$ ;  $\varphi$  is faithful if  $\varphi(x) = 0$  implies  $x = 0$ , normal if it commutes with the sup operation, semi-finite when  $\varphi(x) < +\infty$  on a  $\sigma$ -strongly dense subset of  $M_+$ . Throughout this paper, a weight means a semi-finite normal one.

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