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A SHORTENED PROOF OF SOBOCIŃSKI'S THEOREM CONCERNING A RESTRICTED RULE OF SUBSTITUTION IN THE FIELD OF PROPOSITIONAL CALCULI

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Sobociński [1] proves that in certain circumstances axiomatized systems of the propositional calculus having the rule of simultaneous substitution are not weakened in their deductive power by restricting the application of the substitution rule to the axioms alone. In this paper, a shortened proof of the same result will be presented employing induction on the length of proof sequences. As in Sobociński's proof, it is shown how a proof sequence employing the unrestricted rule may be uniquely and constructively replaced by a proof to the same effect employing only the restricted rule. The proof here draws upon Sobociński's notation and on his proof for certain key steps.

Theorem *If* $\mathbf{T}$ *is an axiom system in the propositional calculus which contains*

(1) *a binary connective* C *among its primitive signs*
(2) *the rule of detachment in regard to* C, R_2
(3) *the rule of simultaneous substitution*, R_1
(4) *an axiom set* A,

and if $\{a_1, \ldots, a_m\}$ *is a finite sequence of axioms and* $\{a_1, \ldots, a_m, b_1, \ldots, b_n\}$ *constitutes a proof sequence in* $\mathbf{T}$ *of* b_n *employing only* R_1 *and* R_2, *then that proof sequence may be replaced by a proof sequence in* $\mathbf{T}$ *of* b_n *which restricts the applications of* R_1 *to* $\{a_1, \ldots, a_m\}$.

Proof: By induction on the length of proof sequences. Call n the "length" of the proof sequence $\{a_1, \ldots, a_m, b_1, \ldots, b_n\}$. Also, where $\{a_1, \ldots, a_m, \ldots, b\}$ is a proof sequence of b in $\mathbf{T}$, $\{a_1, \ldots, a_m\}$ will at times be represented by "α", the rest of the proof sequence by "β_b", and the entire proof sequence by "$\alpha; \beta_b$".

Base Step: $n = 1$. Then the theorem holds directly.

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