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ENUMERATING QUASIPLATONIC CYCLIC GROUP ACTIONS

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ABSTRACT. It is an open problem to determine the number of topologically distinct ways that a finite group can act upon a compact oriented surface X of genus $g(X) \geq 2$. We provide an explicit answer to this problem for special classes of cyclic groups and illustrate our results with detailed examples.

1. Introduction. A consequence of a resolution to the Nielsen realization problem, see [**10**], is that there is a one-to-one correspondence between conjugacy classes of finite subgroups of the mapping class group $\mathcal{M}_\sigma$ of a compact oriented surface of genus σ and the topological equivalence classes of finite groups of homeomorphisms which can act on such a surface. This correspondence has motivated a detailed study of classes of topological group actions, and, in particular, an attempt to classify or enumerate the different ways a group G can act topologically on a surface X of genus $\sigma \geq 2$, see for example, [**2, 3, 6, 7**] where Abelian groups are considered, and [**13, 14**] for other examples. In general, the problem of enumerating classes of topological group actions for arbitrary σ is highly computational and depends very much upon how G acts on X as well as the general structure of G. Indeed, the known results even for very simple classes of groups such as Abelian groups are very technical, and so a general classification for arbitrary groups seems unlikely. Moreover, for an arbitrary G and σ, even answering the simple question, "does G act on a surface of genus σ," is typically non-trivial. These observations motivate a study of classes of topological group actions of structurally simple groups acting in a relatively simple way where it may be possible to derive extremely explicit enumeration formulas as a stepping stone to studying more complicated group actions.

The technical enumeration formulas derived in [**3**] for elementary Abelian group actions of low rank illustrate how difficult it is to obtain

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