COVERING TRANSFORMATIONS AND UNIVERSAL FIBRATIONS

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0. Introduction

The purpose of this paper is to derive the well known formula for the covering transformation group of a covering space by using universal fibrations in the theory of Hurewicz fibrations.

A Hurewicz fibration $p : E \to B$ is a map which has the homotopy covering property for all spaces. We shall assume that B is connected. It is possible to classify the collection of Hurewicz fibrations over CW-complexes up to fibre homotopy equivalence, by means of the universal fibration.

Given any space F, there exists a universal Hurewicz fibration

$$p_\infty : E_\infty \to B_\infty ,$$

with fibre the homotopy type of F, such that any Hurewicz fibration with fibre the homotopy type of F and base space B, a CW-complex, is fibre homotopically equivalent to a Hurewicz fibration induced by a map from B to B_∞. In addition, the homotopy classes of maps from B to B_∞ are in one-to-one correspondence with the fibre homotopy equivalence classes of Hurewicz fibrations with fibre the homotopy type of F and base space B. Also, B_∞ is a CW-complex.

This was first proved for F a compact CW-complex by Stasheff in [8], and later for F a CW-complex by Allaud [1], and for any space F by Dold [2].

Let $p : E \to B$ be a Hurewicz fibration with fibre F. Then there is a map $k : B \to B_\infty$ such that the fibration induced by k is of the same fibre homotopy type as $p : E \to B$. We call k a *classifying map* of $p : E \to B$.

With every fibration $p : E \to B$, there is a group $\mathcal{L}(E)$ which depends on the fibre homotopy equivalences $f : E \to E$. Let $\{f\}$ be the equivalence class of all fibre homotopy equivalence $g : E \to E$ which are homotopic to f by a homotopy h_t such that h_t is a fibre homotopy equivalence for each t. Then define multiplication by $\{f\} \cdot \{g\} = \{f \circ g\}$. This multiplication defines a group $\mathcal{L}(E)$ called the group of *fibre homotopy equivalences*. This group was classified by the author, [5], in terms of B_∞ and the classifying map $k : B \to B_\infty$ corresponding to the fibre space E. We record the result below.

THEOREM. *Let $L(B, B_\infty)$ be the space of continuous maps from B to B_∞ with the compact-open topology. Then $\mathcal{L}(E) \cong \pi_1(L(B, B_\infty); k)$.*

A covering map $p : \tilde{X} \to X$, where $\tilde{X}$ is a covering space, is the earliest example of a Hurewicz fibration. The group of covering transformations is precisely the group of self homotopy equivalences, $\mathcal{L}(\tilde{X})$. Let H be the sub-

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