ILLINOIS JOURNAL OF MATHEMATICS
Volume 31, Number 4, Winter 1987

SMALL SOLUTIONS OF CONGRUENCES OVER ALGEBRAIC NUMBER FIELDS

BY

TODD COCHRANE

1. Introduction

Let R be the ring of integers in a number field K, A be a nonzero ideal in R and $f_1(\mathbf{x}), \dots, f_k(\mathbf{x})$ be homogeneous polynomials in n variables over R. In this paper we obtain small solutions to the system of congruences

$$f_1(\mathbf{x}) \equiv \cdots \equiv f_k(\mathbf{x}) \equiv 0 \pmod{A}, \tag{1.1}$$

the notion of smallness being given two interpretations, as indicated in Lemma 2.

The problem of finding small solutions of congruences has received considerable attention in the case where R is the set of rational integers. For instance, Schinzel, Schlickewei and Schmidt [6, Theorem 1] have shown that for any positive integer m and quadratic form $Q(\mathbf{x})$ over $\mathbf{Z}$ in $n \geq 3$ variables, there is a nonzero solution $\mathbf{x}$ of the congruence

$$Q(\mathbf{x}) \equiv 0 \pmod{m} \tag{1.2}$$

such that $\max|x_i| < m^{1/2+1/2(n-1)}$. Using the same method of proof, Heath-Brown [4, Theorem 2] has shown that if $n = 4$, m is an odd prime and $\det Q$ is a square (mod m), then (1.2) has a nonzero solution with $\max|x_i| < m^{1/2}$.

In this paper we generalize the geometric method of Schinzel et al [6] to algebraic number fields and apply it in turn to systems of linear forms, quadratic forms and forms of higher degree.

We wish to thank our thesis advisor Donald J. Lewis under whom most of this work was conducted, and Hugh L. Montgomery for his comments on the writing of this paper. We also wish to thank Wolfgang M. Schmidt for helping us detect the limitations of this method for forms of degree > 2.

Received February 3, 1986.