WEAK MINIMAL GENERATING SET REDUCTION THEOREMS FOR ASSOCIATIVE AND LIE ALGEBRAS

BY

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It is often difficult to obtain results for Lie algebras over arbitrary fields because the study of Lie algebras over fields of characteristic two and three as well as finite fields usually poses special problems. An attempt was made to develope methods using minimal generators which would be as independent as possible of the nature of the ground field. This led to the author's thesis[1] from which the present paper has been prepared.

Our theorems on Lie algebras essentially use the Jacobi identity only to show that certain subalgebras of Lie algebras are ideals. Hence these results for Lie algebras do not depend on the ground field. This fact also explains why analogous theorems hold for associative algebras. Several propositions determining the structure of certain quotient Lie algebras required a slightly more explicit use of the Jacobi identity. These results were not obtained for fields of characteristic two.

To derive the results of this paper only certain properties of minimal generating sets were used. We single these out by the following definition. A set S of elements of an algebra A weak minimally generates A, abbreviated S w.m.g. A, if

(1) S generates A as an algebra
(2) S consists of linearly independent elements of A
(3) No proper subspace of the vector space spanned by S generates A.

It is now possible to summarize the main results we obtain. Suppose S w.m.g. A and T is a non-empty subset of S. Let B be the subalgebra generated by $S - T$ and C the vector space spanned by T. Now, assume A is the direct sum as vector spaces of B and C and denote the projection of A onto C with respect to this decomposition by P. Then C becomes an algebra with multiplication $*$ defined by $c * c' = P(cc')$ for all $c, c' \in C$. The structure of C with this multiplication is determined. Next for each $c \in C$, $P(bc) = \beta_l(b)c$ and $P(cb) = \beta_r(b)c, b \in B$, where β_l and β_r are linear functionals from B into F. If A is either an associative or Lie algebra with dim $C \geqq 2$ then the kernel $\beta_l \cap$ kernel β_r is an ideal in A. In a slightly different direction, if B is an ideal in A and the base field is infinite then A is the direct sum as vector spaces of the subalgebra generated by all products of elements of B

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[1] *The structure of Lie algebras with large minimal generating sets*, Mathematics Ph.D. thesis, University of Notre Dame, Indiana, June 1964, written under the direction of Hans Zassenhaus.