LOCALLY CONNECTED SPACES AND THEIR COMPACTIFICATIONS[1]

BY

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Introduction

This paper arose from the following problem: to find conditions under which a locally connected, rim-compact Hausdorff space H has a locally connected Hausdorff compactification. This proves to be the case (Theorem 4.2) if and only if at most finitely many of the components of H are compact. In trying to find a simple proof, the authors realized that a systematic and simple approach to the theory of locally connected spaces can be obtained by stressing—even more than Wilder [13]—the use of quasicomponents. We use, among other notions (e.g. "paddedness") the property of being quasilocally connected at a point [13, p. 40], and observe that the useful Proposition 1.7 holds for quasicomponents but not for components. The quasicomponent approach leads naturally to the result that in a connected, compact Hausdorff space, the property "components coincide with quasicomponents on every open subset" is equivalent to local connectedness (Theorem 3.3). (Recall that components and quasicomponents coincide on every *closed* subset of *any* compact Hausdorff space.)

To ask for a reasonable necessary and sufficient condition that a locally connected completely regular Hausdorff space have a locally connected Hausdorff compactification seems hopeless. However, it is known [6] that *every* compactification of such a space is locally connected if and only if the space is pseudo-compact. We readily obtain a proof of this theorem, and add some corollaries.

Example 5.3 seems to be of interest. Here we exhibit a subspace S of Euclidean three-space which is the union of a countable number of pairwise disjoint closed intervals, each nowhere dense in S, but S is nevertheless connected and locally connected.

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1. Components and quasicomponents

1.1. DEFINITIONS. Let p be a point in the space X. If the subset S of X is both open and closed in X, we say S is *clopen* in X.

The *component* of p in X is the maximal connected subset of X containing p.

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