NORMAL COMPLEMENTS OF CARTER SUBGROUPS

BY

SURINDER K. SEHGAL AND WILLIAM A. MCWORTER, JR.

(To Professor A. E. Ross on his 60th birthday, August 24, 1966)

Carter [3] has shown that if U is a subgroup of a finite group G that satisfies

(a) U is nilpotent,
(b) $N_G(U) = U$,
(c) U is a Hall subgroup of G,
(d) the Sylow subgroups of U are regular;

then G has a normal subgroup N satisfying $UN = G$ and $U \cap N = \{1\}$, (i.e., U has a normal complement in G).

In case G is solvable, a nilpotent, self-normalizing subgroup U is called a Carter subgroup of G. It follows easily from Lemma 1 below that if U is a Carter subgroup of a solvable group G and U has a normal complement in G, then G has property P (for a definition of property P see Carter [1]). We will show in this paper that if U is a subgroup of a finite solvable group G having property P and satisfying properties (a), (b), and (c) of Carter's theorem stated above, then U has a normal complement in G.

We will show by means of an example that condition (c) in the statement of our theorem is necessary. We will also show that there exists groups G having property P where the Sylow subgroups of Carter subgroups are not regular. The last construction also shows the existence of infinitely many finite solvable groups containing a given nilpotent group as a Carter subgroup.

The following lemma is proved in [5].

LEMMA. *A finite solvable group G has property P if and only if it has a subgroup U satisfying*

(a) *U is maximal nilpotent,*
(b) *there exists a normal subgroup $N \neq \{1\}$ of G with $U \cap N \subseteq Z(G)$ or $U = G$,*
(c) *property* (b) *is satisfied by the image of U in any homomorphic image. Furthermore U is a Carter subgroup of G.*

THEOREM. *If U is a subgroup of a finite solvable group G having property P such that*

(a) *U is nilpotent,*
(b) *$N_G(U) = U$,*
(c) *U is a Hall subgroup of G,*

then U has a normal complement in G.

Received March 31, 1967.