ALMOST PERIODIC SEMIGROUPS IN TRANSFORMATION GROUPS

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Introduction

If a measure preserving flow is defined on a measure space X of finite measure, then the flow is incompressible, and it follows that almost all the points of X are recurrent (Poincaré recurrence theorem; see [5, p. 10]). W. H. Gottschalk and G. A. Hedlund [3], [4, chapter 8], and C. W. Williams [10] have exploited similar ideas in topological dynamics, and J. D. Baum [1] has also done this in a more general setting. A. Khintchine [8] has shown that if m is a proability measure defined on X, then

$$\lim_{T-S\to\infty} \frac{1}{T-S}\int_S^T m(A \cap At)\,dt > m^2(A)$$

for a measure preserving flow, where A is any measurable set. (See also E. Hopf [7, p. 40].) If the measure is a Borel measure not vanishing on open sets and the flow is continuous, it follows that the flow is regionally almost periodic (under Gottschalk and Hedlund's definition [4]). The purpose of this paper is to "topologize" notions related to the measure preserving property and investigate their relationship to almost periodicity properties.

In the first section it is shown that in many phase spaces it is sufficient for some almost periodicity properties to establish the corresponding property with respect to a replete semigroup in the transformation group, but that this is not true for regional almost periodicity. The following section investigates boundedness, incompressibility, and dissipative properties.

1. S-almost periodicity

1.1 *Notation.* Let (X, T, π) or (X, T) denote a transformation group. We assume that T is generative, and we let S denote a replete semigroup in T (see [4] for definitions). We assume that X is a Hausdorff space. Whenever we assume that X is a uniform space, we assume that it has the uniform topology and write small Greek letters for elements of the uniformity.

1.2 DEFINITION. A subset B of T is called *S-syndetic* if there exists a compact subset K of T for which $BK \supset S$. This definition differs from that of Gottschalk and Hedlund [4, p. 63]. All S-almost periodicity properties which we will study are now defined as in [4, 3.13] where "S-syndetic" replaces "admissible."

1.3 *Remark.* The following statements are pairwise equivalent:
(1) The transformation group (X, T) is S-almost periodic.

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