ON COLLAPSIBLE BALL PAIRS[1]

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One of the essential parts of Zeeman's proof [20], [21] to show that ball pairs B^q, B^s, $q - s \geq 3$, were unknotted was to show that B^q collapses to B^s. For $q - s = 2$, it is well known that there exist ball pairs B^q, B^{q-2}, such that B^q, B^{q-2} are knotted but B^q collapses to B^{q-2} for $q \geq 4$. For $q = 1, 2, 3$, it is known that B^q, B^{q-1} is unknotted and hence B^q collapses to B^{q-1} [5]. We say B^q, B^s is a *collapsible ball pair* if B^q collapses to B^s. In this paper we examine ball pairs B^q, B^{q-1} for $q \geq 4$ with regards to collapsibility. It is known that B^4, B^3 is unknotted iff B^4, B^3 is a collapsible ball pair; however, it is unknown whether there exist knotted B^q, B^{q-1} for $q \geq 4$. We show that for $q \geq 6$, every B^q, B^{q-1} is a collapsible ball pair and give some necessary and sufficient conditions that B^q collapses to B^{q-1} for $q = 4, 5$. We also characterize all ball pairs B^5, B^4.

Terminology and definitions will be as in [20] except as follow. By a manifold, we mean a locally Euclidean, separable metric space. When referring to combinatorial manifolds and piecewise linear maps we shall always use the adjectives combinatorial and piecewise linear. Let M be an orientable manifold; by bdry M we mean the boundary of M with the induced orientation; by int M, the interior of M; by M^- we mean M with its orientation reversed. By Cl X, we mean the closure of X.

THEOREM 1. *Let B^n, B^{n-1} be a ball pair with $n \geq 6$; then B^n collapses to B^{n-1}.*

1. Proof of Theorem 1 for $n \geq 7$

Let N be an admissible regular neighborhood of B^{n-1} in B^n [20; Chap. VII, p. 67]. Then $N \cap \text{bdry } B^n$ is a regular neighborhood of bdry B^{n-1} in bdry B^n. It was shown in [8] that

$$\text{Cl}\,(\text{bdry } B^n - (N \cap \text{bdry } B^n))$$

is the union of two disjoint combinatorial $(n - 1)$-cells, say $S_1 \cup S_2$. Similarly, Cl (bdry $N - (N \cap \text{bdry } B^n)$) is the union of two disjoint combinatorial $(n - 1)$-cells, say $T_1 \cup T_2$, indexed so that $S_i \cap T_i \neq \emptyset$, $i = 1, 2$. Then each $S_i \cup T_i$ is a combinatorial $(n - 1)$-sphere. Hence by considering the double of B^n, it follows from [4], [15] that each $S_i \cup T_i$ bounds a topological

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