LOCALIZATION THEOREMS IN HARMONIC ANALYSIS ON R^n

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1. The theory of Rajchman and Zygmund on localization properties of trigonometric series (see for instance [6, pp. 330–344, 363–370]) has been extended to the multi-dimensional case by Berkovitz, Gosselin and Shapiro in [1], [2] and [4] (for a brief account of some of their results see [5, p. 83–85]). They discuss the localization problem for summability kernels of a very special type and their methods are very closely linked to the particular properties of the kernels. Our aim is to present a method, in one dimension basically the same as Zygmund's method in [6] (the proof of his Theorem 9.6), and which is applicable to a larger class of kernels.

The method can be applied to R^n as well as to Z^n. We choose to present it on R^n for the reason that the study on R^n contains all the essential ideas which are needed for the corresponding study on Z^n, on the other hand, it is more general due to the non-compactness of the character group of R^n. In order to facilitate a comparison of our results on R^n with the previously established results on Z^n we have, in the last section, formulated a localization theorem for Bochner-Riesz summability.

In the following we make freely use of the basic concepts of Fourier analysis of tempered distributions, as presented in any standard text-book on the subject.

2. Let q be a positive and continuous function on R^n, such that

$$p(x) = \sup_{y \in R^n} q(x + y)/q(y) \tag{1}$$

is finite for all $x \in R^n$, bounded on compact sets and $O(|x|^A)$, for some positive number A, as $|x| \to \infty$. Obviously p is a Borel measurable positive function. We introduce the space M_p of Borel measures μ on R^n with a finite norm

$$\| \mu \|_p = \int_{R^n} p(x) \, | d\mu(x) |,$$

the space B_q of Borel measurable functions g on R^n with a finite norm

$$\| g \|_q = \int_{R^n} | g(x) | \, g(x) \, dx,$$

and finally the space B^q of Borel measurable functions f on R^n with a finite norm

$$\| f \|^q = \operatorname{ess\,sup}_{x \in R^n} | f(x) | / q(-x),$$

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