A CHARACTERIZATION OF THE ALTERNATING GROUP OF DEGREE ELEVEN

BY
TAKESHI KONDO

Introduction

The main purpose of the present paper is to prove the following theorem.

THEOREM B. *Let G be a finite group satisfying the following conditions:*

(1) *G has no normal subgroup of index 2, and*

(2) *G contains an involution z_0 such that $C_G(z_0)$ is isomorphic to the centralizer of an involution in the center of an S_2-subgroup of A_{11}, the alternating group of degree eleven.*

Then G is isomorphic to A_{11}.

Let D be a 2-group of order 2^7 which is isomorphic to a wreath product of a dihedral group of order 8 by a group of order 2. An S_2-subgroup of A_{11} is isomorphic to D. Further, there are infinitely many simple groups with an S_2-subgroup isomorphic to D, namely $LF_4(q)$ ($q \equiv 3 \bmod 8$) and $U_4(q)$ ($q \equiv 5 \bmod 8$). So the present paper contains detailed discussions of a finite group with an S_2-subgroup isomorphic to D, which are more than necessary for the proof of Theorem B (cf. footnote 2)). The results are summarized in Theorem A of §4.

Notation:

$\mathrm{ccl}_X(x)$	a conjugate class in a group X containing x
$\langle \cdots \rangle$	a group generated by ...
X'	the commutator subgroup of a group X
$[x, y]$	$x^{-1}y^{-1}xy$
x^y	$y^{-1}xy$
$x \sim y$ in X	x is conjugate to y in a group X
$J(X)$	the Thompson subgroup of X (cf. [7])
$C_X^*(x)$	$\langle y \in X \mid y^{-1}xy = x^{\pm 1} \rangle$.
A_n (S_n)	the alternating (symmetric) group of degree n

The other notations are standard.

1. Preliminaries

(1.0) Let X be a finite group and S be an S_2-subgroup of X. Let K be a subset of S which is an intersection of S with a conjugate class of X. An element x of K is called an extreme element if $|C_S(x)| \geqq |C_S(y)|$ for any $y \in K$. The following is due to R. Brauer [1, p. 308].

(1.1) LEMMA. *If x is an extreme element of K, $C_S(x)$ is an S_2-subgroup of $C_X(x)$. Moreover, for every element y of K, there exists an isomorphism θ*

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