SOME REMARKS ON S. WEINGRAM: ON THE TRIANGULATION OF A SEMISIMPLICIAL COMPLEX [8]

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In this note we use the terminology and notation of [8].

1. Intention

Weingram's paper is concerned with a proof of the following theorem [8, Theorem 1.1].

THEOREM 1. *Let X be a semisimplicial complex and $|X|$ its geometric realization (in the sense of* [6] *or* [7]). *Then there is a functor D from the category of semisimplicial complexes and semisimplicial maps to that of ordered simplicial complexes and weak order-preserving maps, a transformation of functors $\lambda : D \to 1$, and, for each X, a map $t_x : |DX| \to |X|$ such that*

(i) *t_x is a homeomorphism (and therefore a triangulation of $|X|$);*

(ii) *t_x defines a subdivision of the CW complex $|X|$; and*

(iii) *$|\lambda(X)|$ is homotopic to t_x by a homotopy F such that for each cell $|e|$ of $|DX|$, F maps $|e| \times I$ into the smallest cell $|x|$ of $|X|$ which contains $t_x(|e|)$.*

The statement of this theorem is correct, also the idea of the proof given at the end of §1 in Weingram's paper. But in the details there are several mistakes which shall be corrected in the following.

2. On the barycentric subdivision functor

Weingram needs the following

THEOREM 2. *For any semisimplicial complex X, there is a homeomorphism $t : |\mathrm{Sd}\, X| \to |X|$ identifying $|\mathrm{Sd}\, X|$ with a subdivision of the CW-complex $|X|$* [8, Proposition 2.5].

In order to prove this theorem, Weingram wants—analogous to M. G. Barratt [1]—to subdivide $|X|$ by a modified star-subdivision process. To do so, he has to choose in each face of a topological simplex which is the source of the realization of the characteristic map of a simplex of X an interior point—called pseudo-barycenter—and to subdivide these topological simplices by starring. Having done this Weingram states without proof that there is a "consistent subdivision of $|X|$". However in general this is impossible, as it is shown by the counterexample described at the end of [4]. Nevertheless the statement of theorem 2 is correct, proofs can be found in [2] and [4], which also contain proposition 2.6 of [8].

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