CELL COVERINGS AND RESIDUAL SETS OF CLOSED MANIFOLDS

BY

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1. Introduction and preliminaries

Very simple arguments can be combined with recent results of Summerhill and Pedersen to improve the known bounds for the minimal number of open n-cells required to cover an n-dimensional topological manifold. There are two basic methods of proof, both of which depend on the notion of *residual set*, a concept invented by Doyle and Hocking [1] to mean the complement of a dense open n-cell in a closed topological manifold.

The terminology used in this report follows the usage in [6], except where an explicit reference is cited. In what follows, M denotes a closed topological manifold of dimension n. Using the language of residual sets, the recent results which were referred to above are the following:

SUMMERHILL'S THEOREM. *Let $0 \leq k \leq n - 3$. If M is k-connected then there is a strong Z_{k-1}-set which is residual in M.*

Actually, Summerhill proves the converse as well. For the proof, see [8]; for facts about Z_m-sets, also see [7].

PEDERSEN'S THEOREM. *Let $3 \leq k$ and $6 \leq n$. If M is k-connected then there is a polyhedron of dimension $n - k$ locally tamely embedded in M which is residual in M.*

The proof is immediate from corollary 3 of [5] and the topological version of the Tubular Neighborhood Theorem [2].

2. The de Morgan approach

The salient point of this method is the observation that M can be covered by r *dense* open n-cells if and only if M has r residual sets whose common intersection is empty. (This is just de Morgan's law. The reader might enjoy the easy exercise of proving that the product of two spheres can always be covered by three open cells, using this idea.) In order to apply this idea to the problem at hand, we need another result of Summerhill [7], a less technical rendition of which is

GENERAL POSITION THEOREM. *Let X and Y be "tame" closed subsets of E^n having dimensions p and q, respectively. Then arbitrarily close to the identity there exists a self-homeomorphism h of E^n such that $X \cap h(Y)$ has dimension at most $p + q - n$.*

Received July 7, 1975.