M-PROJECTIVE AND STRONGLY M-PROJECTIVE MODULES

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Introduction

Given a module M over a ring R, G. Azumaya [1] introduced the dual notions of M-projective and M-injective modules. These concepts have actually led M. S. Shrikhande to a study of hereditary and cohereditary modules [5]. More recently Azumaya, Mbuntum and the present author obtained necessary and sufficient conditions for the direct sum $\oplus_{\alpha \in J} A_\alpha$ of a family of modules to be M-injective [2]. While R-injective modules are the same as injective modules over R, the class of R-projective modules in the sense of Azumaya in general is larger than the class of projective R-modules. In this paper we introduce the notion of a strongly M-projective module and the associated notion of a strong M-projective cover. Next we investigate strong M-projective covers. We show that if every module possesses a strong M-projective cover then $R/\mathfrak{A}(M)$ is (left) perfect, where $\mathfrak{A}(M)$ is the annihilator of M. If $R/\mathfrak{A}(M)$ is perfect, we show that every R-module A with $t_M(A) = 0$ possesses a strong M-projective cover, where

$$t_M(A) = \{x \in A \mid f(x) = 0 \text{ for all } f \in \operatorname{Hom}(A, M)\}.$$

Another application of the ideas here is the result that if $\mathfrak{A}(M) = 0$, then an R-module B is strongly M-projective iff B is projective. In particular if R is (left) perfect and $\mathfrak{A}(M) = 0$, then an R-module B is M-projective iff B is actually projective. Since $\mathfrak{A}(R) = 0$, we can regard this result as a generalization of the "known" result that when R is perfect an R-module is R-projective iff it is projective. It will be interesting to characterise the rings with the property that R-projective modules are the same as the projective modules over R.

1. Preliminaries

Throughout this paper R denotes a ring with $1 \neq 0$, R-mod the category of unital left modules. All the modules we deal with are unital left modules. M denotes a fixed object in R-mod. We recall briefly the concepts of M-projective and M-injective modules introduced by G. Azumaya and state two results due to him [1].

DEFINITION 1.1. A module P is called M-projective if given any eipmorphism $\phi: M \to N$ and any $f: P \to N$, there exists a $g: P \to M$ such that $\phi \circ g = f$.

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