STACKING TRANSFORMATIONS AND DIOPHANTINE APPROXIMATION

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Introduction

The stacking method (see [1] and [6, Section 6]) has been used with great success in ergodic theory to construct a wide variety of examples of ergodic transformations (see, for example, [1], [4], [5], [6], [10]). However, very little is known in general about the class $\mathcal{S}$ of transformations obtained by the stacking method using single stacks. In particular there is no simple characterization of the class $\mathcal{S}$. In [1], the following question is raised: is every transformation with simple spectrum an $\mathcal{S}$-transformation? (The converse is true by [2, Theorem 1].) As a particular case the following question is also raised: is the translation by an irrational number α in $[0, 1)$ an $\mathcal{S}$-transformation? In Section 1 of this paper we answer this question affirmatively for α in a set E of Lebesgue measure 1, as well as giving a partial negative result for α in E^c. We also consider certain products of translations. Section 2 is concerned with giving an explicit stacking construction having $e^{2\pi i\alpha}$ as an eigenvalue. We show this is possible for almost all α, and for all α in E^c. All these results depend on various conditions connected with the goodness of approximation by rationals of the irrationals involved and we prove several results asserting the existence of irrationals satisfying these conditions.

The methods of this paper can also be used to show that the examples considered in [8], Sections 8 and 9, belong to $\mathcal{S}$ thereby also furnishing examples of transformations with continuous spectrum and mixed continuous and discrete spectrum respectively (other than examples actually constructed by the stacking method). We shall not give the proofs here.

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Section 0

All measure spaces $(X, \mathcal{F}, \mu)$ will be isomorphic to $[0, 1]$ with Borel sets and Lebesgue measure. A transformation (automorphism) of $(X, \mathcal{F}, \mu)$ is an invertible, bimeasurable, measure preserving mapping of X onto X. A partition of X is a finite collection of mutually disjoint elements of $\mathcal{F}$. If $\{P_n\}$ is a sequence of partitions, $P_n \to \varepsilon$ means $\mu(A \Delta P_n(A)) \to 0$ for all $A \in \mathcal{F}$, where $P_n(A)$ denotes any union of atoms of P_n such that $\mu(P_n(A) \Delta A)$ is minimal. If T is a trans-

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