COMPLETELY REDUCIBLE ACTIONS OF CONNECTED ALGEBRAIC GROUPS ON FINITE-DIMENSIONAL ASSOCIATIVE ALGEBRAS

BY

JOHN BRENDAN SULLIVAN[1]

Introduction

Let G be a group which acts completely reducibly by algebra automorphisms on a finite-dimensional associative K-algebra A, which is separable modulo its radical.

When the characteristic of K is zero, Mostow showed in [4], using the representation theory of reductive algebraic groups, that there is a G-invariant separable subalgebra of A complementary to the radical (a G-invariant Wedderburn factor).

Taft in [5] conjectured that there is a G-invariant Wedderburn factor when characteristic K is $p \neq 0$.

In this paper, we verify the conjecture when K is perfect and the image of G in the algebraic group of algebra automorphisms of $A \otimes_K \bar{K}$ has connected closure [see Theorem 1].

Relevant facts about separable algebras may be found in [1, Section 72], and about algebraic groups in [3].

Let A be a finite-dimensional associative algebra over a field K, with radical R. Suppose that A/R is a separable algebra and that S is a separable subalgebra of A complementary to R. S will be called a Wedderburn factor of A. Let $p: A \to R$ be the projection of the sum $A = S \oplus R$ onto the factor R; let $\pi: A \to A/R$ be the quotient map.

Let G be a group which acts completely reducibly on A by algebra automorphisms. Write gb for the image of $b \in A$ under $g \in G$.

All mappings are K-linear.

Section 1

Throughout this section, $R^2 = (0)$.

Let V be a G-invariant subspace of A complementary to R, and let $h: A \to R$ be the projection of the sum $A = V \oplus R$ onto the factor R. Let $f = h \mid S$: $S \to R$.

We introduce a second action $(*)$ of G on A which stabilizes S. The two actions coincide if and only if S is G-invariant under the original action.

Received January 16, 1975.

[1] Supported in part by a National Science Foundation grant.