ILLINOIS JOURNAL OF MATHEMATICS
Volume 24, Number 2, Summer 1980

INTERPOLATION SETS AND EXTENSIONS OF THE GROTHENDIECK INEQUALITY

BY

RON BLEI[1]

The key step in the proof of the Grothendieck inequality is an "integral representation" of the inner product in a Hilbert space (see Theorem 2.3 in [1], for example):

THEOREM. *There is a compact abelian group G, a constant $K > 0$, and a function Φ: $l^2 \to L^\infty(G)$ so that*

(i) *for all $x \in l^2$, $\|\Phi(x)\|_\infty \leq K\|x\|_2$,*
and
(ii) *for all $x, y \in l^2$, $(x, y) = (\Phi(x) * \Phi(\bar{y}))(0)$ ($*$ denotes convolution).*

A natural task is to extend the above theorem and design an analogous representation for the dual action between l^p and l^q, where p and q are conjugate exponents. This is what we do in this paper. The present work could be viewed as a postscript to [1], and, indeed, methods here are modifications of those used in [1].

We employ basic notation and facts of commutative harmonic analysis as presented and followed in [5]. Γ, as usual, will be a discrete group and $G = \Gamma^\wedge$ will denote its compact dual group. In the first section, work will be performed in the framework of $\otimes \mathbf{Z}_2 = \Omega$, the (compact) direct product of $\mathbf{Z}_2$, and $\oplus \mathbf{Z}_2 = \hat{\Omega}$, its (discrete) dual group, the direct sum of $\mathbf{Z}_2$. Throughout, $E = \{r_n\}_{n=1}^\infty \subset \hat{\Omega}$ will denote the system of Rademacher functions realized as characters in $\hat{\Omega}$. In Section 1, we extend the notions of $\Lambda(2)$ and Sidon sets: $F \subset \Gamma$ is an $L(p)$ set, $1 < p \leq 2$, if for every $\phi \in l^p(F)$ there is $f \in L^\infty(G)$ so that $\hat{f} = \phi$ on F, and $\hat{f} \in l^p$. Analogously, F is an $S(q)$ set, $2 < q \leq \infty$, if for every $\phi \in l^q(F)$ there is $\mu \in M(G)$ so that $\hat{\mu} = \phi$ on F and $\hat{\mu} \in l^q$. These notions lead to an integral representation of the dual action between l^p and l^q (Theorem 1.3), and to an extension of the classical Grothendieck inequality (Corollary 1.4). Deserving a study for its own sake, the $L(p)$ property is briefly examined in the second section, where a class of $L(p)$ sets is obtained as a subclass of $\Lambda(q)$ sets of a certain type (Theorem 2.2). We conclude with some questions.

Received March 29, 1978.

[1] The author was partially supported by a National Science Foundation grant.