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COMPLEMENT THEOREMS BEYOND THE TRIVIAL RANGE[1]

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1. Introduction

By a well known theorem of Chapman [2], if X and Y are Z-sets in the Hilbert cube Q, then X and Y have the same shape (abbreviated $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$) if and only if $Q - X$ is homeomorphic with $Q - Y$. In recent years there has been a great deal of interest in finite-dimensional analogues of this result, the principal aim being to find conditions on compacta $X, Y \subset E^n$ such that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ if and only if $E^n - X \cong E^n - Y$. Thus far all results along this line have required either that the dimensions or fundamental dimensions of X and Y lie at most in the trivial ($[n/2] - 1$) range with respect to n [3], [7], [8], [11], [17], [20] or that $\mathrm{Sh}(X)$ and $\mathrm{Sh}(Y)$ have particularly nice representatives, such as spheres, manifolds, or finite complexes [4], [11], [12], [13], [15], [21]. It is our purpose to present here a theorem in (fundamental) codimension four. We are able to go beneath the trivial range in ambient dimension by assuming appropriate connectivity conditions on the embedded compacta; these conditions allow us to replace general position arguments which suffice in the trivial range by ones using engulfing. Our main result is as follows.

THEOREM A. *Let X and Y be r-shape connected continua in E^n of fundamental dimension at most k and satisfying ILC, where*

$$n \geq \max(2k + 2 - r, k + 3, 5).$$

Then $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ *implies* $E^n - X \cong E^n - Y$. *The converse holds if* $n \geq k + 4$.

Nowak [14] has shown that if X is a finite dimensional approximatively 1-connected compactum and $\check{H}^i(X) = 0$ for $i > k$, then $Fd(X) \leq k$. This fact along with Theorem A yields the following.

THEOREM B. *Let X and Y be r-shape connected continua in E^n satisfying ILC and such that $\check{H}^i(X) = 0 = \check{H}^i(Y)$ for $i > k$, where*

$$n \geq \max(2k + 2 - r, k + 3, 5)$$

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