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ON THE EXPONENTIAL STABILITY OF SET-VALUED DIFFERENTIAL EQUATIONS

BY

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Introduction

In this paper, we consider necessary and sufficient conditions for the (local) exponential stability of set-valued differential equations defined in $\mathbf{R}^k$.

To this aim, we introduce the concepts of local spectrum $\Sigma_p(F)$ and numerical range $N_p(F)$ for a set-valued Lipschitz-like map F defined in a neighborhood of a point $p \in \mathbf{R}^k$. We show that $\Sigma_p(F) \subset N_p(F)$ and that the condition $N_p(F) \subset \mathbf{R}^- = \{x \in \mathbf{R}, x < 0\}$ ensures the exponential stability of the set-valued differential equation $\dot{x} \in F(x)$. On the other hand, we show that exponential stability implies $\Sigma_p(F) \subset \mathbf{R}^-$. Section 3 contains some applications of our results to the stability of nonlinear control systems.

We note that for linear maps the concept of local spectrum and numerical range are well known [1]. For nonlinear maps such concepts were introduced by Furi and Vignoli [6]. For set-valued maps, the definition of asymptotic spectrum was proposed in [3]. An approach to stability problems for set-valued functions can be found in [2] and in [4].

1. Definition and preliminary results

DEFINITION 1. We consider $\mathbf{R}^k$ with the standard norm and set-valued maps

$$F\colon U \to 2^{\mathbf{R}^k}\backslash\{\phi\} = S(\mathbf{R}^k),$$

where U is open in $\mathbf{R}^k$. Such a map is continuous if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that $\|x - y\| < \delta$ implies $h(F(x), F(y)) < \varepsilon$, where h denotes the Hausdorff distance on $S(\mathbf{R}^k)$.

Henceforth we shall only consider continuous set-valued maps F on $\mathbf{R}^k$ where $F(x)$ is a compact set for every x.

For $p \in \mathbf{R}^k$, let $Q_p(\mathbf{R}^k)$ be the set of all such continuous set-valued functions defined in some neighborhood of p for which

(I) $F(p) = \{0\}$,

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