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ON THE COHOMOLOGY OF THE LIE ALGEBRA OF FORMAL VECTOR FIELDS PRESERVING A FLAG

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1. Let

$$\mathscr{A}_{n,r} = \left\{ \sum_{i=1}^{r} f_i(x_1, \ldots, x_r) \frac{\partial}{\partial x_i} + \sum_{i=r+1}^{n} f_i(x_1, \ldots, x_n) \frac{\partial}{\partial x_1} \,\middle|\, f_i\text{-formal power series in the variables concerned.} \right\}$$

and

$$\mathscr{A}_r = \left\{ \sum_{i=1}^{r} f_i(x_1, \ldots, x_r) \frac{\partial}{\partial x_i} \,\middle|\, f_i\text{-formal power series in } x_1, \ldots, x_r \right\}$$

The cohomology groups of $\mathscr{A}_r$ were studied by Gelfand and Fuks [4]. In this paper we prove that $\mathscr{A}_{n,r}$ is r-connected: $H^i(\mathscr{A}_{n,r}\mathbf{R}) = 0$ for $0 < i \leq r$.

In this context Professor A. Haefliger asked the author whether

$$H^i(\mathscr{A}_{n,r}, \mathbf{R}) \simeq H^i(\mathscr{A}_r, \mathbf{R}) \quad \text{for } i \leq 2n \text{ (canonically).}$$

Here we prove this isomorphism for $i \leq n - r$ only (Theorem 3.6). The method of this paper is not powerful enough to answer Haefliger's question for $i > n - r$.

The method of proof is essentially that employed by M. Jacques Vey [10] in proving a vanishing theorem for the cohomology of the formal Poisson algebra.

We describe below how the cohomology groups of $\mathscr{A}_{n,r}$ ($\mathscr{A}_r$) are related to the characteristic classes of a flag of foliations (a foliation). For more details see [3] and [1].

Let M^m be a smooth manifold of dimension m. A flag of smooth foliations of codimensions r, n ($r \leq n$) is a pair of foliations $\mathscr{F}_r$, $\mathscr{F}_n$ on M of codimensions r, n respectively such that the leaves of $\mathscr{F}_n$ are contained in the leaves of $\mathscr{F}_r$. Let ν_r be the normal bundle of $\mathscr{F}_r$ and let

$$\nu_{n-r} = \frac{\text{normal bundle of } \mathscr{F}_n}{\text{normal bundle of } \mathscr{F}_r}.$$

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