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A NON-REMOVABLE SET FOR ANALYTIC FUNCTIONS SATISFYING A ZYGMUND CONDITION

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1. Introduction

A complex-valued function f defined on the complex plane $\mathbf{C}$ satisfies a Lipschitz condition of order α, $0 < \alpha \leq 1$, if there exists a constant $C(f)$ such that

$$|f(z+h) - f(z)| \leq C(f)|h|^{\alpha}$$

for all complex z and h. This condition is obviously stronger than

$$|f(z+h) + f(z-h) - 2f(z)| \leq C(f)|h|^{\alpha}$$

which does not necessarily imply the continuity of f. When $\alpha = 1$, the latter condition is usually called the Zygmund condition. We shall denote the classes of bounded continuous functions which satisfy the above conditions respectively by Lip_α and Λ_α. If $0 < \alpha < 1$, it is well known (see [5, Chap. V, Section 4]) that Lip_α and Λ_α are identical but $\mathrm{Lip}_1 \subsetneqq \Lambda_1$.

We shall call a compact subset E of $\mathbf{C}$, a removable set for analytic functions of class Lip_α, resp. Λ_α, provided that every function in Lip_α, resp. Λ_α, which is analytic in $\mathbf{C} \setminus E$ has analytic extension to the entire plane. Dolženko [1] proved that E is removable for analytic functions of class Lip_α, $0 < \alpha < 1$, if and only if E has $(1+\alpha)$-dimensional measure zero. In [6] we showed that this result is also true for the case $\alpha = 1$. Thus the removable sets for analytic functions of class Λ_1 must also have zero $dx\,dy$-measure.

In this paper we shall construct a compact set E of zero $dx\,dy$-measure and a probability Borel measure μ, supported on E, such that its Cauchy transform

$$\hat{\mu}(z) = \int \frac{d\mu(\zeta)}{\zeta - z}$$

belongs to Λ_1. Since $\hat{\mu}(z) = -1/z + \cdots$ at ∞, $\hat{\mu}$ cannot be entire.

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