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AMENABLE HYPERGROUPS

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1. Introduction

The purpose of this paper is to initiate a systematic study of amenable hypergroups. The theory of hypergroups was initiated by Dunkl [13], Jewett [28] and Spector [49] and has received a good deal of attention from harmonic analysts. Hypergroups naturally arise as double coset spaces of locally compact groups by compact subgroups. In [42], Pym also considers convolution structures which are close to hypergroups. A fairly complete history is given in Ross' survey article [45].

Throughout, K will denote a hypergroup with a left Haar measure λ. It is still unknown if an arbitrary hypergroup admits a left Haar measure, but all the known examples such as commutative hypergroups [50] and central hypergroups [24] do.

Let $L_\infty(K)$ be the Banach space of all bounded Borel measurable functions on K with the essential supremum norm. A left invariant mean on $L_\infty(K)$ is a positive linear functional of norm one, which is invariant under left translations by elements in K. K is said to be amenable if there is a left invariant mean on $L_\infty(K)$.

Section 2 consists of notations used throughout this paper.

In Section 3, we give examples and discuss stability properties of amenable hypergroups. In contrast to the result of Granirer [21] and Rudin [47] for the group case, we exhibit a class of commutative hypergroups K for which every invariant mean on $L_\infty(K)$ is topologically invariant.

In Section 4, Reiter's condition (P_1) is shown to characterize amenability of hypergroups. It is also shown that, if a hypergroup satisfies (P_2), then it has property (P_1), and that the converse is not true in general. This is again in contrast to the group case.

In [33], Lau introduced and studied a class of Banach algebras which include $L_1(K)$. He called such algebras F-algebras. He extended several important characterizations of amenable locally compact groups to left

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