

SOME RESEARCH PROBLEMS ABOUT ALGEBRAIC DIFFERENTIAL EQUATIONS II

BY
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Dedicated to my son and daughter, Mark and Natasha

In 1983, I published (see [RUB-I]) a collection of research problems about ADE's. Since then, some of the problems have been solved, some have resisted all attacks, and a number of new ones have been generated. In this paper, which I have titled as Part II of the original one, I bring the project as up to date as I can. We refer the reader to the "Explanatory section" of Part I for some of the definitions and notation, as well as for specific problems described there. I also pose a number of new problems. As in Part I, I surround the new problems with descriptive material, including background and partial solutions, where they are available.

I repeat here the closing paragraph of Part I.

"In summary, these are some aspects of the subject as I have expressed them in problems. I hope not too many of them turn out to be embarrassingly easy. At any rate, I have had going making limited progress on a few. I hope people who find complete or partial solutions will communicate them to me and to others. Finally, I hope I have shown that, just a little off the beaten track in differential equations, there lies a fascinating vein of mathematics."

1. Problems that have been (partially) solved

(Numbers of problems in this section refer to Part I of this paper [RUB-I].) Of course, Problems 1 and 3 were already solved in Part I. We recall:

PROBLEM 2. *What kind of gaps can the power series of a solution of an ADE have?*

Several partial solutions to this problem were later given in [LIR-I]. The most complete of these results goes as follows. Let $\{n_k\}$ be an increasing

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