ILLINOIS JOURNAL OF MATHEMATICS
Volume 36, Number 4, Winter 1992

NATURALLY REDUCTIVE RIEMANNIAN S-MANIFOLDS

BY

A.J. LEDGER

1. Introduction

Riemannian k-symmetric spaces and, more generally, Riemannian regular s-manifolds have been studied by several authors and the general theory is now well established. All such manifolds are homogeneous and the associated canonical connection [5] is determined by the symmetry tensor field S of type $(1, 1)$ which is derived from s. Riemannian locally regular s-manifolds can then be defined either in terms of s or, more usefully for our purpose, in terms of the invariance of certain tensor fields under the action of S as a field of tangent space endomorphisms. Thus, as well as the first order condition that ∇S should be S-invariant, where ∇ is the Riemannian connection, one requires the second order conditions that $\nabla^2 S$ and the curvature tensor field R are S-invariant and then the third order condition that ∇R is S-invariant.

For a regular s-manifold, the homogeneous Riemannian structure can be shown to be naturally reductive [5] if and only if S satisfies the additional first order condition $(\nabla_{(I-S)^{-1}X} S)S^{-1}X = 0$ for all vector fields X. In turn, this condition can be applied to define the notion of naturally reductive for locally regular s-manifolds and one might then ask whether such a first order condition can be used to simplify the higher order conditions given above. The simplest example is afforded by a Riemannian locally symmetric space which can be defined either by local 2-symmetries or by the single condition $\nabla R = 0$. In this case $S = -I$ so the above tensor conditions are trivial except for ∇R being S-invariant, that is $\nabla R = 0$. Moreover, this condition reduces to

$$(\nabla_X R)(X, JX, X, JX) = 0$$

for the Hermitian case. A less trivial case arises with locally 3-symmetric spaces. These are almost Hermitian with almost complex structure J satisfying

$$S = -\frac{1}{2}I + \frac{\sqrt{3}}{2}J$$

Received December 7, 1990
1991 Mathematics Subject Classification. Primary 53B20, 53C25, 53C30, 53C35, 53C40.

© 1992 by the Board of Trustees of the University of Illinois
Manufactured in the United States of America