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ARRAY NONRECURSIVE DEGREES AND LATTICE EMBEDDINGS OF THE DIAMOND

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1. Introduction

In [16], Slaman solved Barry Cooper's diamond problem by constructing an r.e. degree $\mathbf{a} \neq \mathbf{0}$ that was not the top of a diamond in the Δ_2 degrees. That is, for all Δ_2 degrees $\mathbf{b}, \mathbf{c}$, if $\mathbf{b} \cup \mathbf{c} = \mathbf{a}$ and $\mathbf{b}|\mathbf{c}$ then $\mathbf{b} \cap \mathbf{c} \neq \mathbf{0}$. The proof of this result was a very complex argument involving "three jumps" of nonuniformity. Earlier work of Cooper [3], Posner [15] and Epstein [9] showed that such $\mathbf{a}$ could not be high, indeed if $\mathbf{b}$ is high then the degrees $\leq \mathbf{b}$ are complemented.

Naturally the question arises: exactly what r.e. degrees are tops of diamonds? Fejer [10] has proved that if $\mathbf{a}$ is an r.e. degree that is not the top of a diamond then $\mathbf{a}$ is low$_2$. (In fact, Fejer showed that if $\mathbf{a}$ is a degree that is non-GL$_2$ then $\mathbf{a}$ is the top of a diamond.) Subsequently, Slaman pointed out a possible definition of "low$_2$ r.e.": perhaps all low$_2$ r.e. degrees are bounded by degrees that are not tops of diamonds in the Δ_2 degrees.

The first goal of the present paper is to answer Slaman's question negatively. In fact a consequence of our results is that there are low r.e. degrees not bounded by any r.e. degree that is not the top of a diamond.

In [6], Downey, Jockusch and Stob initiated the study of a new class of r.e. degrees called the *array nonrecursive r.e. degrees*. We review the definition of this class in §2, but for our purposes here it suffices to remark that it is a natural class of degrees which arise from arguments which need 'multiple permissions'. In [6] it is shown that this class contains members of low degree, is closed upwards, and contains all non-low$_2$ r.e. degrees.

Our result is:

(1.1) THEOREM. *Suppose* $\mathbf{a}$ *is array nonrecursive. Then* $\mathbf{a}$ *is the top of a diamond in the* Δ_2 *degrees.*

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