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MIXED AUTOMORPHIC FORMS ON SEMISIMPLE LIE GROUPS

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1. Introduction

Let Γ be a discrete subgroup of $PSL(2, \mathbb{R})$ and let $\chi \colon \Gamma \to SL(2, \mathbb{R})$ be a homomorphism of groups. Then both Γ and $\chi(\Gamma)$ operate on the Poincaré upper half plane $\mathcal{H}$ by linear fractional transformations. We assume that there is a holomorphic map $\omega \colon \mathcal{H} \to \mathcal{H}$ satisfying $\omega(gz) = \chi(g)\omega(z)$ for all $g \in \Gamma$ and $z \in \mathcal{H}$. Given a pair of nonnegative integers k and l with k even, a holomorphic function $f \colon \mathcal{H} \to \mathbb{C}$ satisfying the condition

$$f(gz) = (cz + d)^k (c_\chi \omega(z) + d_\chi)^l f(z)$$

for all $z \in \mathcal{H}$ and

$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma, \qquad \chi(g) = \begin{pmatrix} a_\chi & b_\chi \\ c_\chi & d_\chi \end{pmatrix} \in SL(2, \mathbb{R})$$

is a holomorphic mixed automorphic form of one variable of type (k, l) associated to Γ, ω and χ if f satisfies an additional condition of regularity at the cusps of Γ (see [16]). Certain types of such mixed automorphic forms occur naturally as holomorphic forms of the highest degree on elliptic varieties which are fiber varieties over an arithmetic variety with generic fiber a product of elliptic curves (cf. [10], [15]). Holomorphic mixed automorphic forms of several variables were also introduced in [17] and [18], and it was proved that a certain class of such automorphic forms can be interpreted as holomorphic forms on some families of abelian varieties over an arithmetic variety.

The purpose of this paper is to describe mixed automorphic forms in the setting of representations of semisimple Lie groups following such descriptions for the usual automorphic forms initiated by Selberg and Langlands (e.g., see [3], [4], [9], [21]). More specifically, we define mixed automorphic forms on semisimple Lie groups which generalize holomorphic mixed automorphic forms, and construct Poincaré series and Eisenstein series for such automorphic forms.

2. Mixed automorphic forms on a semisimple Lie group

First, we shall review the definition of the usual automorphic forms on semisimple Lie groups (e.g., see [3], [4], [5], [6]). Let G be a semisimple Lie group, and let $\mathfrak{g}$ be

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