ON A CLASS OF DOUBLY TRANSITIVE PERMUTATION GROUPS[1]

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Let Ω be the set of symbols $1, \cdots, m+1$. Let $\mathfrak{G}$ be a doubly transitive permutation group on Ω in which no nontrivial permutation leaves three symbols fixed. Such a group $\mathfrak{G}$ will be called a Zassenhaus group.

On the structure of Zassenhaus groups Feit [4] proved recently the following elegant theorem: Let $\mathfrak{G}$ be a Zassenhaus group of degree $m+1$, which contains no normal subgroup of order $m+1$. Then m must be a power of a prime number: $m = p^e$. Let $\mathfrak{M}$ be a Sylow p-subgroup of $\mathfrak{G}$, and let $\mathfrak{M}'$ be the commutator subgroup of $\mathfrak{M}$. Then the index of $\mathfrak{M}'$ in $\mathfrak{M}$ must be smaller than $4q^2$, where q is the order of the subgroup $\mathfrak{Q}$, which consists of all the permutations leaving each of the symbols 1 and 2 fixed. Moreover if $\mathfrak{M}$ is abelian, then $q \geqq (m-1)/2$.

Now the purpose of this paper is to prove the following.

THEOREM. *If m is odd, then $\mathfrak{M}$ must be abelian.*

1. In the following $\mathfrak{G}$ denotes always a Zassenhaus group of even degree $m+1$, which contains no normal subgroup of order $m+1$. Let Γ_i $(i = 0, 1, 2)$ be the set of all the permutations in $\mathfrak{G}$, each of which fixes just i symbols of Ω. Then according to our assumptions on $\mathfrak{G}$ we obtain the following decomposition of $\mathfrak{G}$ into its mutually disjoint subsets: $\mathfrak{G} = \Gamma_0 + \Gamma_1 + \Gamma_2 + \{1\}$, where 1 is the identity element of $\mathfrak{G}$.

Since $\mathfrak{G}$ is doubly transitive, $\mathfrak{G}$ possesses an irreducible character $\mathbf{B}$, whose values can be written as follows:

$$(1) \qquad \mathbf{B}(X) = \begin{cases} m & \text{for } X = 1, \\ 1 & \text{for } X \in \Gamma_2, \\ 0 & \text{for } X \in \Gamma_1, \\ -1 & \text{for } X \in \Gamma_0. \end{cases}$$

2. Let $\mathfrak{G}_1$ be the subgroup of $\mathfrak{G}$, which consists of all the permutations leaving the symbol 1 fixed. Then we can choose an $\mathfrak{M}$ in the theorem of Feit in the following way: $\mathfrak{M}$ is a normal subgroup of $\mathfrak{G}_1$ and satisfies the conditions that $\mathfrak{G}_1 = \mathfrak{M}\mathfrak{Q}$ and $\mathfrak{M} \cap \mathfrak{Q} = 1$. Now we assume that

(2.1) $\mathfrak{M}$ *is not abelian.*

Therefore the purpose of our proof is to derive a contradiction from this

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