INDECOMPOSABLE REPRESENTATIONS

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1. Introduction

Let Λ be a finite-dimensional algebra over a field K. By a Λ-module we shall mean always a finitely generated left Λ-module on which the unity element of Λ acts as identity operator. It is well known that the Krull-Schmidt theorem holds for Λ-modules: each module is a direct sum of indecomposable Λ-modules, and these summands are uniquely determined up to order of occurrence and Λ-isomorphism. Thus the problem of classifying Λ-modules is reduced to that of finding the isomorphism classes of indecomposable Λ-modules. We denote the set of these by $M(\Lambda)$.

A central problem in the theory of group representations is that of determining a set of representatives of $M(\Lambda)$ for the special case where $\Lambda = KG$, the group algebra of a finite group G over the field K. A definitive answer can be given when the characteristic of K does not divide the group order $[G:1]$; in this case KG is semisimple, all indecomposable modules over KG are irreducible, and a full set of non-isomorphic minimal left ideals of KG constitute a set of representatives of $M(KG)$. For the case where the characteristic of K is p ($p \neq 0$), Higman [6] has proved the following remarkable result: *$M(KG)$ is finite if and only if the p-Sylow subgroups of G are cyclic.* If such is the case, Higman obtained an upper bound on the number of elements of $M(KG)$. A best possible upper bound was later obtained by Kasch, Kupisch, and Kneser [5].

We shall attempt to elucidate Higman's theorem by considering in detail the special case where G is an abelian p-group, and K a field of characteristic p. We shall exhibit some new classes of indecomposable modules. However we shall show that the problem of computing $M(KG)$, in case G is not cyclic, is at least as difficult as a classical unsolved problem in matrix theory.

It should be pointed out that the question of determining all representations of a p-group in a field of characteristic p has been extensively treated by Brahana [1, 2, 3] from a somewhat different viewpoint. There is consequently a certain amount of overlapping between his results and ours, but we have thought it best to make this paper completely self-contained.

2. C-algebras

Inasmuch as we shall need to consider, together with modules over an algebra Λ, also modules over sub- and quotient-algebras of Λ, we cannot re-

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