MEAN-L-STABLE SYSTEMS

BY

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1. Introduction

Let X be a compact metric space with metric ρ, and let T be a homeomorphism of X onto itself. The pair (X, T) will be called a *compact system.*

In this paper we shall be concerned with compact systems which are mean-L-stable, as defined in Section 4. The definition of mean-L-stable systems is due to Fomin [3]. Mean-L-stable systems were also discussed briefly by Oxtoby in [7]. The theorems he obtained will be quoted at appropriate places in this paper.

We adopt the following notations. If E is a set, χ_E denotes its characteristic function, and E' denotes its complement (when the containing space is understood.) If E is a subset of X, its closure is denoted by $\bar{E}$.

If E is a set of integers, let

$$\delta_k(E) = (2k + 1)^{-1}\textstyle\sum_{j=-k}^{k} \chi_E(j).$$

The *upper density* of E, $\delta^*(E)$, is defined by

$$\delta^*(E) = \limsup_{k\to\infty} \delta_k(E),$$

and the *lower density of* E, $\delta_*(E)$, is defined by

$$\delta_*(E) = \liminf_{k\to\infty} \delta_k(E).$$

If $\delta_*(E) = \delta^*(E)$, their common value is called the *density of* E, and is denoted by $\delta(E)$.

2. Measure theoretic preliminaries. The theory of Kryloff and Bogoliouboff

A *Borel measure* on X is a finite measure on the algebra of all Borel subsets of X. A Borel measure μ is *normalized* if $\mu(X) = 1$. An *invariant Borel measure* on (X, T) is a Borel measure μ on X such that if E is a Borel subset of X, then $\mu(E) = \mu(ET)$. It is known [7, (2.1)] that any compact system admits at least one normalized invariant Borel measure. A Borel subset E on X is said to have *invariant measure zero* (*invariant measure one*) provided $\mu(E) = 0$ $(\mu(E) = 1)$ for every normalized invariant Borel measure μ on (X, T).

Received July 28, 1958.

[1] The material of this paper is taken from a Ph.D. thesis submitted to the University of Pennsylvania. The author is indebted to Professor W. H. Gottschalk for many valuable suggestions. During part of the time the author was writing this paper he was a research assistant on an Air Force Contract.