

LINEAR SPACES WITH A COMPACT GROUP OF OPERATORS¹

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Introduction

This paper is devoted to the establishment of several results of a rather general nature concerning a class of locally convex topological linear spaces that have compact abelian groups of operators. The class includes many of the spaces that occur in analysis as function spaces on compact groups or as completions of such spaces.

Section 1 is devoted to some necessary preliminary results concerning Fourier analysis in the spaces in question. The results are hardly original but do not seem to be in the literature in the form needed here. In Section 2 the formal topology is introduced. For the spaces that occur in analysis this topology agrees with that of formal (i.e., termwise) convergence of Fourier or power series. Our main result is that in the spaces under consideration a convex, invariant subset, closed in the original topology, remains closed in the much weaker formal topology. Several applications are given. The next section is devoted to a result establishing the equivalence of several types of continuity for linear transformations that commute with the group operations. In Section 4 we associate to each space of the type under consideration its G -dual (which will in general be smaller than its ordinary dual space) and prove that the association is reflexive, that is, that a space is canonically isomorphic to the G -dual of its G -dual.

We have restricted ourselves to compact abelian groups for the sake of simplicity and in order to avoid as many computations as possible. All of our results, with the appropriate modifications, remain valid for arbitrary compact groups.

In the following we assume a knowledge of the rudiments of the theory of locally convex topological linear spaces as found for example in [1]. Also needed are some of the simplest facts concerning vector valued integration of continuous functions (contained for example on pp. 79–89 of [2]); the fact of which we make most crucial use is that the integral of a continuous function with respect to a positive measure of mass 1 lies in the convex closure of its range.

I wish to express my indebtedness to H. Mirkil. Most of the ideas in this paper grew out of conversation with him.

1. Fourier analysis in G -spaces

Let G be a compact abelian group and A a complete locally convex topological linear space over the complex numbers. Assume that there is associated

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