LINEAR SYSTEMS OF FIRST AND SECOND ORDER DIFFERENTIAL EQUATIONS WITH PERIODIC COEFFICIENTS

BY

J. K. HALE

Consider the system of linear differential equations

$$\tag{1} \begin{cases} u'' + A_1(\lambda)u = \lambda f_1(u, v, w, u', v', t, \lambda), \\ v'' + A_2(\lambda)v = \lambda f_2(u, v, w, u', v', t, \lambda), \\ \qquad\quad w' = \lambda f_3(u, v, w, u', v', t, \lambda), \end{cases}$$

where λ is a real parameter,

$$u = (y_1, \cdots, y_\nu), \qquad v = (y_{\nu+1}, \cdots, y_\mu) \qquad w = (y_{\mu+1}, \cdots, y_n),$$

$$f_1 = (f_1^*, \cdots, f_\nu^*), \qquad f_2 = (f_{\nu+1}^*, \cdots, f_\mu^*), \qquad f_3 = (f_{\mu+1}^*, \cdots, f_n^*),$$

$A_1(\lambda) = \operatorname{diag}(\sigma_1^2, \cdots, \sigma_\nu^2)$, $A_2(\lambda) = \operatorname{diag}(\sigma_{\nu+1}^2, \cdots, \sigma_\mu^2)$, and the vector functions f_1, f_2, f_3 are *linear* functions of u, v, w, u', v'. The coefficients in these linear functions are real, periodic functions of t of period $T = 2\pi/\omega$, L-integrable in $[0, T]$, analytic in λ, and have mean value zero. Further, suppose that each $\sigma_j(\lambda)$, $j = 1, 2, \cdots, \mu$, is a real positive analytic function of λ with $\sigma_j(0) \pm \sigma_h(0) \neq m\omega$, $j \neq h$, $j, h = 1, 2, \cdots, \mu$, $\sigma_h(0) \neq m\omega$, $h = 1, 2, \cdots, \mu$, $m = 1, 2, \cdots$. Systems of type (1) for $|\lambda|$ small have recently been extensively investigated by a method which has been successively developed by L. Cesari, J. K. Hale and R. A. Gambill for both linear [1, 3, 4, 6, 8] and weakly nonlinear differential systems [2, 5, 7]. Most of the previous work has been concerned with systems of type (1) without the third vector equation, i.e., with systems of second order equations. The aim of the present paper is to prove a theorem concerning the boundedness of the AC (absolutely continuous) solutions of (1). By applying the same methods, the following theorem is proved:

THEOREM. *If*

(α) $f_1(u, -v, w, -u', v', -t, \lambda) = f_1(u, v, w, u', v', t, \lambda)$,

(β) $f_2(u, -v, w, -u', v', -t, \lambda) = -f_2(u, v, w, u', v', t, \lambda)$, *and*

(γ) $f_3(u, -v, w, -u', v', -t, \lambda) = -f_3(u, v, w, u', v', t, \lambda)$,

then for $|\lambda|$ *sufficiently small, all the AC solutions of* (1) *are bounded in* $(-\infty, +\infty)$.

This theorem generalizes some previous results of the author [8] for systems of linear equations of type (1) where the third vector equation did not

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