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BOOK REVIEWS

M. FRANK NORMAN, *Markov Processes and Learning Models.* Academic Press, New York, 1972, xiii + 274 pp. $ 15.00

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This book by Frank Norman is devoted to random processes suggested by certain learning models. It adds in a remarkable way and with a strong personal flavor to the family of existing books dealing with this subject.

As the author states himself (page xi) "no attempt is made to establish the psychological utility of these models." More a probabilist then a psychologist, as the author qualifies himself, he develops a very beautiful mathematical theory motivated by stochastic models for learning. The results, many of which are due to the author, are, at present, enriching the theory of a certain class of stochastic processes; their practical utility is to be tested by the future.

All the examples of learning models (l.m.'s) quoted in the introduction (Chapter 0) have the following structure (page 12). At the beginning of trial n, the subject is characterized by his *state of learning* X_n, which takes on values in a *state space* X. On this trial, an *event* E_n occurs, in accordance with a probability distribution $p(X_n, G) = P(E_n \in G \mid X_n)$ over subsets G of an *event space* E. This, in turn, effects a transformation $X_{n+1} = u(X_n, E_n)$ of state.

Now let $(X, \mathscr{B})$ and $(E, \mathscr{G})$ be measurable spaces, let p be a stochastic kernel on $X \times \mathscr{G}$, and let u be a transformation of $X \times E$ into X, measurable with respect to $\mathscr{B} \times \mathscr{G}$ and $\mathscr{B}$. Following M. Iosifescu (see, e.g., M. Iosifescu and R. Theodorescu, *Random Processes and Learning*, Springer (1969) page 63), we call $((X, \mathscr{B}), (E, \mathscr{G}), p, u)$ a (homogeneous) *random system with complete connections* (r.s.c.c.). Further, we call a sequence $\{X_n, E_n\}_{n\geqq 0}$ of random vectors on a probability space $(\Omega, \mathscr{F}, P)$ an *associated stochastic process* if X_n and E_n take on values in $(X, \mathscr{B})$ and $(E, \mathscr{G})$, respectively, $X_{n+1} = u(X_n, E_n)$ and $P(E_n \in G \mid X_n, E_{n-1}, \cdots) = p(X_n, G)$ a.s., for each $G \in \mathscr{G}$.

On page 24 Norman writes: "The concept of r.s.c.c. may be regarded as a generalization and formalization of the notion of a stochastic l.m. Thus we will often call such a system a l.m. or simply a *model*." Consequently, for studying special cases of l.m.'s, first general r.s.c.c.'s or, in Norman's terminology, l.m.'s, are examined. In other words, Norman's main aim is to study r.s.c.c.'s = l.m.'s. It follows that Norman's results will form an interesting and important contribution to the theory of r.s.c.c.'s, which originated in 1935 with a paper by O. Onicescu and G. Mihoc.

Further, consider the following processes: $\{X_n\}_{n\geqq 0}$, $\{E_n, X_{n+1}\}_{n\geqq 0}$, $\{X_n, E_n\}_{n\geqq 0}$, and $\{E_n\}_{n\geqq 0}$ generated by an r.s.c.c. It is easily seen that the first three processes