$$\text{var } u_p(0) \sim \left\{ \frac{(2q+1)(2q-1)\cdots 1}{2^q q!} \right\}^2 \sigma^2/n, \tag{13}$$

where q is $\frac{1}{2}p$ when p is even and $\frac{1}{2}(p-1)$ when p is odd. In the region of extrapolation, when $|x|$ is large (12.2) gives

$$\text{var } u_p(x) \doteqdot (2p+1)\{(2p)!/2^p p!^2\}^2 x^{2p} \sigma^2/n.$$

The deviations from these formulae when n is not large have been discussed and tabulated [4].

4. Comparison of the two methods. In the central part of the range the uniform spacing method gives a smaller variance than the minimax variance method. An asymptotic expansion of (13) using Stirling's factorial approximation shows that the ratio of the variances is roughly $2/\pi$. This ratio increases steadily with $|x|$, and at the ends of the range the variance for the uniform spacing method exceeds that for the minimax variance method by a factor $p+1$, while in the region of extrapolation this factor approaches $2+p^{-1}$. The crossover points for the two variance curves occur at ± 0.58 for the quadratic and ± 0.72 for the cubic. Thus over most of the region of interpolation the advantage lies with the uniform spacing method, but at the extremes of the region of interpolation and in the region of extrapolation the advantage lies decidedly with the minimax variance method.

Fig. 1 shows the shape of the two variance curves in the region of interpolation for the second and third degree polynomials. Since the curves are symmetrical about the origin of x, only half of each curve is drawn.

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CONDITIONS THAT A STOCHASTIC PROCESS BE ERGODIC[1]

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In his work on statistical inference on stochastic processes, Grenander has pointed out ([2], p. 257) that "the concept of metric transitivity seems to be

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